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The Fermi-sea-like limit of the composite fermion wave function

O. Ciftja[a]

Ames Laboratory, Iowa State University, Ames, Iowa 50011, USA

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Abstract. The experimentally observed filling factors of the fractional quantum Hall effect can be described in terms of the composite fermion wave function of the Jastrow-Slater form $\Psi^{\rm CF}_{\nu=p/(2mp+1)} = \hat{P}_{\rm LLL}\prod_{j<k}^{N}(z_j - z_k)^{2m}\Phi_p(B^*)$ fully projected into the lowest Landau level. The Slater determinant of the above composite fermion wave function represents the filled Landau levels of composite fermions evaluated at the corresponding reduced magnetic field. For a system of fermions studied in the thermodynamic limit, we prove that in the even-denominator-filled state limit (when the number of filled Landau levels of composite fermions becomes infinite), the above composite fermion wave function exactly transforms into the Rezayi-Read Fermi-sea-like wave function $\Psi^{\rm Fermi}_{\nu=1/(2m)} = \hat{P}_{\rm LLL}\prod_{j<k}^{N}(z_j - z_k)^{2m}{\rm Det}\{\varphi_{\mathbf{k}}(\mathbf{r})\}$ constructed by attaching $2m$ flux quanta to the Slater determinant of two-dimensional free fermions at the density corresponding to that filling. We study the composite fermion wave function and its evolution into the Fermi-sea-like wave function for a range of filling factors very close to the even-denominator-filled state.

PACS. 71.10.Pm Fermions in reduced dimensions (anyons, composite fermions, Luttinger liquid, etc.) – 73.40.Hm Quantum Hall effect (integer and fractional) – 71.27.+a Strongly correlated electron systems; heavy fermions

1 Introduction

The fractional quantum Hall effect [1] (FQHE) results from a strongly correlated incompressible liquid state [2,3] formed at special uniform densities $\rho(\nu)$ of a two-dimensional (2D) electronic system which is subjected to a very strong perpendicular magnetic field $\mathbf{B}$. The dominant sequence of fractional Hall states occurs when the filling of the lowest Landau level (LLL) is $\nu = p/(2mp+1)$, where $p = 1, 2, \ldots$ and $m = 1, 2, \ldots$ are integers. Much of the theoretical work on the FQHE is based on the study of the properties of a 2D fully spin-polarized (spinless) system of N interacting electrons embedded in a uniform positive background. The electrons with charge $-e$ ($e > 0$) and mass m_e are considered confined in the $x - y$ plane of area Ω and subjected to a magnetic field, $\mathbf{B} = (0, 0, B)$ which is generated from the symmetric gauge vector potential $\mathbf{A}(\mathbf{r}) = (-By/2, Bx/2, 0)$. We will consider the thermodynamic limit of an infinite system defined as the limit of N electrons in a sample of area Ω, where N and Ω go to infinity with the density kept constant.

The many-electron system is described by the Hamiltonian $\hat{H} = \hat{K} + \hat{V}$, where $\hat{K}$ is the kinetic energy operator

$$\hat{K} = \frac{1}{2m_e}\sum_{j=1}^{N}\left[-{\rm i}\hbar\boldsymbol{\nabla}_j + e\mathbf{A}(\mathbf{r}_j)\right]^2 \qquad (1)$$

and

$$\hat{V} = \sum_{j<k}^{N} v(|\mathbf{r}_j - \mathbf{r}_k|) - \rho(\nu)\sum_{j=1}^{N}\int {\rm d}^2 r\; v(|\mathbf{r}_j - \mathbf{r}|) + \frac{\rho(\nu)^2}{2}\int {\rm d}^2 r_1 \int {\rm d}^2 r_2\; v(|\mathbf{r}_1 - \mathbf{r}_2|) \qquad (2)$$

is the total electron-electron, electron-background and background-background interaction potential, where $v(|\mathbf{r}_j - \mathbf{r}_k|) = e^2/(4\pi\epsilon_0\epsilon|z_j - z_k|)$ is the interaction potential, $z_j = x_j + {\rm i}y_j$ is the location of the j-th electron in complex coordinates and ϵ is the dielectric constant of the background.

It has become clear in recent years that many essential features of the FQHE can be understood straightforwardly in terms of a new kind of particle, called a composite fermion (CF), which is a bound state of an electron and an even number of vortices of the many-body quantum wavefunction [4,5] formed at the electronic densities $\rho(\nu) = \nu/(2\pi l_0(B)^2)$ where $l_0(B) = \sqrt{\hbar/(eB)}$ is the electronic magnetic length. The fundamental property of the CF-s is that they experience a reduced effective field, $B^* = B(1 - 2m\nu)$ so that the quantum liquid of strongly correlated electrons at B is equivalent to a quantum liquid of weakly interacting CF-s at B^*. Since the degeneracy of each Landau level is proportional to the magnetic field, the degeneracy N_s^* of each CF Landau level will be smaller than the corresponding N_s for the electrons and

[a] e-mail: ciftja@chronus.ameslab.gov

will be given by $N_s^* = N_s(1 - 2m\nu)$. As a result the effective filling factor of CF-s will be an integer number $\nu^* = p$ and will correspond to stable electronic filling factors $\nu = \nu^*/(2m\nu^* + 1)$, where $\nu^* = p$ is the number of filled CF Landau levels.

There are two calculational schemes based on the intuitive physics above. One constructs explicit wave functions [4] while the second scheme employs a Chern-Simons (CS) field theory [6] approach to investigate the CF state. Although the two schemes are based on the same physics, a precise quantitative relationship between them is not clear. The trial wave function $\Psi^{\rm CF}_{\nu=p/(2mp+1)}$ given by

$$\Psi^{\rm CF}_{\nu=p/(2mp+1)} = \hat{P}_{\rm LLL} \prod_{j<k}^{N} (z_j - z_k)^{2m} \Phi_p(B^*), \qquad (3)$$

where $\Phi_p(B^*)$ is the Slater determinant wavefunction of p filled CF Landau levels, evaluated at the magnetic field shown in the argument and $\hat{P}_{\rm LLL}$ is the LLL projection operator is due to Jain [4]. For the special case of the ground state at $\nu = 1/(2m + 1)$, namely for $p = 1$ the above CF wave function becomes the Laughlin wave function [3], which is already known to be a very accurate representation of the exact ground state at $\nu = 1/3$ and 1/5.

By contrast, the behaviour of such a system in the vicinity of a filling factor with even denominator such as $\nu = 1/(2m)$ is not well understood. At these fillings the typical features of the FQHE are not observed and there is sound experimental evidence [7,8] that these states are Fermi-like compressible states. Based on a CS field approach, Halperin, Lee, and Read [6] (HLR) have proposed a theory of a compressible Fermi-like behaviour and studied within the random-phase approximation (RPA) many physical quantities. The CS formulation of interacting electrons nicely realizes the concept of CF-s, in accordance with the experimental observations, but nevertheless the understanding, and even the evaluation of the RPA still seem to be incomplete [9].

Several studies have shown that the above CF wave function is a valid description for all experimentally observed incompressible FQHE states. Intuitively, one would expect that this description remains valid also for large values of p. In particular, the limit $p \to \infty$ corresponds to the even-denominator-filled state $\nu = 1/(2m)$ which is known to be a compressible Fermi-like state [10].

By studying the system of fermions in the thermodynamic limit, we prove that in the even-denominator-filling limit, the CF wave function of equation (3) exactly transforms into the Rezayi-Read [11] Fermi-sea-like wave function constructed by attaching $2m$ flux quanta to the 2D Fermi sea of free fermions at the corresponding density. In Section 2 we give the proof of this transformation and in Section 3 we present some numerical results obtained by using the CF wave function in a wide range of filling factors around the even-denominator-filled state and by using the Fermi-sea-like wave function at the exact even-denominator-filled state.

2 The composite fermions

The study of strongly correlated systems described by equations (1, 2) constitutes a very difficult problem. The simplest approach to understand the ground-state properties of such a many-body system is to construct a reasonable trial wave function, such as the CF wave function $\Psi^{\rm CF}_{\nu=p/(2mp+1)}$ in order to incorporate the most prominent physical processes at the very beginning. Then the calculation of the energy per particle or other physical quantities of interest depends on the ability to compute exactly the radial distribution function $g_\nu(r_{12})$ which in terms of the trial CF many body wave function is given by

$$g_{\nu=p/(2mp+1)}(r_{12}) = \frac{N(N-1)}{\rho(\nu)^2} \frac{\int {\rm d}^2 r_3 \cdots {\rm d}^2 r_N |\Psi^{\rm CF}_{\nu=p/(2mp+1)}|^2}{\int {\rm d}^2 r_1 \cdots {\rm d}^2 r_N |\Psi^{\rm CF}_{\nu=p/(2mp+1)}|^2}, \qquad (4)$$

and for the system under consideration will depend only on the interparticle spatial distance $r_{12} = |\mathbf{r}_1 - \mathbf{r}_2|$. An exact calculation of the radial distribution function can be achieved only in very special cases and, in general, one should adopt approximate methods to compute it. In this case we note that since the CF wave function $\Psi^{\rm CF}_{\nu=p/(2mp+1)}$ is of Jastrow-Slater form then a natural method to be applied is the Fermi-hypernetted-chain [12–14] theory which, as its Bose counterpart [3,15,16], allows for a realistic evaluation of several quantities for systems of fermions in the thermodynamic limit.

Within the FHNC theory, the radial distribution function $g_\nu(r_{12})$ is expressed as a sum of irreducible cluster diagrams constructed with (i) the "bosonic" bond $h(r_{jk}) = f(r_{jk})^2 - 1$, where $f(r_{jk})^2 = |z_j - z_k|^{4m}$ is the dynamical correlation factor and (ii) the "statistical exchange" bond $l_p(\mathbf{r}_j, \mathbf{r}_k) = \hat{\rho}_p(\mathbf{r}_j, \mathbf{r}_k)/\rho(\nu)$, where $\hat{\rho}_p(\mathbf{r}_j, \mathbf{r}_k)$ is the one-body density matrix corresponding to the dynamically uncorrelated state $\Phi_p(B^*)$ and $\rho(\nu)$ is the electronic (CF) density.

At this stage, the only quantities that specify the Jastrow-Slater many-body wave function are the dynamical correlation and the statistical exchange factor which determine respectively the Jastrow and the Slater part of the wave function. Knowing them, if we were able to sum all the sets of cluster diagrams appearing in equation (4) then in principle we would have been able to carry on an exact calculation.

Unfortunately, it is known that the FHNC technique is intrinsically approximated because there is a set of cluster diagrams (corresponding to the so called *elementary* diagrams) which cannot be fully included in any closed form. Several schemes have been devised to include such cluster diagrams at various levels of approximation. The simplest approximation of totally neglecting the *elementary* diagrams (FHNC/0) leads to reliable results and we adopted it in this paper.

The full FHNC/0 formalism and its application to the Laughlin states has been reported elsewhere [17], here we generalize it for the CF wave function of equation (3) by skipping further details. In order to compute the statistical

exchange factor corresponding to $\Phi_p(B^*)$ we need to know the eigenstates corresponding to the uncorrelated system of CF-s.

For a magnetic field B^* applied in the z direction generated by a symmetric gauge vector potential $\mathbf{A}(\mathbf{r})$, the eigenstates of the ideal Hamiltonian

$$\hat{H}_0 = \frac{1}{2m_e}\left[-\mathrm{i}\hbar\boldsymbol{\nabla} + e\mathbf{A}(\mathbf{r})\right]^2 \tag{5}$$

for the various CF Landau levels $n = 0, 1, 2, \ldots$ are given by

$$\varphi_{n,l}(z) = \frac{1}{\sqrt{2^n n!}} \exp\left[\frac{|z|^2}{4l_0(B^*)^2}\right]\left[2l_0(B^*)\frac{\partial}{\partial z}\right]^n \times \left\{\varphi_{0,l}(z)\exp\left[-\frac{|z|^2}{4l_0(B^*)^2}\right]\right\}, \tag{6}$$

where $l_0(B^*) = \sqrt{\hbar/(eB^*)}$ is the CF's magnetic length and

$$\varphi_{0,l}(z) = \frac{1}{\sqrt{2^l l!}}\left[\frac{z}{l_0(B^*)}\right]^l \varphi_{0,0}(z), \tag{7}$$

$$\varphi_{0,0}(z) = \frac{1}{\sqrt{2\pi l_0(B^*)^2}}\exp\left[-\frac{|z|^2}{4l_0(B^*)^2}\right], \tag{8}$$

where $l = 0, 1, \ldots, (N_s^* - 1)$ is the angular momentum quantum number for the CF-s. The one-body density matrix $\hat{\rho}_p(\mathbf{r}_1, \mathbf{r}_2)$ is given by

$$\hat{\rho}_p(z_1, z_2) = g_s \sum_{n=0}^{p-1}\sum_{l=0}^{N_s^*-1} \varphi^*_{n,l}(z_1)\varphi_{n,l}(z_2), \tag{9}$$

where the spin degeneracy g_s of each CF Landau state is $g_s = 1$, since we are considering fully spin-polarized (spinless) electrons (CF-s). In the above equation, the sum is extended over all occupied CF states, where $\varphi_{n,l}(z)$ and $\varphi^*_{n,l}(z)$ are respectively the single particle wave function and its conjugate and we use complex coordinates z_i instead of the two-dimensional vectors $\mathbf{r}_i$.

Using a standard algebra, one can easily prove that the contribution to $\hat{\rho}_p(z_1, z_2)$ coming from the n-th CF Landau level is

$$\sum_{l=0}^{N_s^*-1}\varphi^*_{n,l}(z_1)\varphi_{n,l}(z_2) = L_n\left(\frac{|z_1 - z_2|^2}{2l_0(B^*)^2}\right) \times \sum_{l=0}^{N_s^*-1}\varphi^*_{0,l}(z_1)\varphi_{0,l}(z_2), \tag{10}$$

where $L_n(x) = \frac{\mathrm{e}^x}{n!}\frac{\mathrm{d}^n}{\mathrm{d}x^n}(x^n\mathrm{e}^{-x})$ are the Laguerre polynomials of order $n = 0, 1, \ldots$. By using the formulas given in equations (7, 8) one can express the second term of equation (10) as

$$\sum_{l=0}^{N_s^*-1}\varphi^*_{0,l}(z_1)\varphi_{0,l}(z_2) = \frac{1}{2\pi l_0(B^*)^2} \times \exp\left[-\frac{|z_1|^2 + |z_2|^2}{4l_0(B^*)^2}\right]\sum_{l=0}^{N_s^*-1}\frac{1}{l!}\left(\frac{z_1^* z_2}{2l_0(B^*)^2}\right)^l. \tag{11}$$

In the thermodynamic limit both the density and the filling factor $\nu = N/N_s$ are kept constant as the number of electrons N and the LL degeneracy N_s go to infinity. Since the degeneracy N_s^* of each CF Landau level is directly proportional to N_s then also $(N_s^* - 1)$ goes to infinity in the thermodynamic limit. As a result the summation over l in equation (11) is extended from 0 to ∞ and one obtains

$$\sum_{l=0}^{N_s^*-1}\varphi^*_{0,l}(z_1)\varphi_{0,l}(z_2) = \frac{1}{2\pi l_0(B^*)^2} \times \exp\left[-\frac{|z_1|^2 + |z_2|^2}{4l_0(B^*)^2}\right]\exp\left[\frac{z_1^* z_2}{2l_0(B^*)^2}\right]. \tag{12}$$

By substituting equation (12) in equation (10), one notes that, in the thermodynamic limit, the one-body density matrix $\hat{\rho}_p(z_1, z_2)$ given from equation (9) depends solely on p and not N_s^* and is given by

$$l_p(r_{12}) = \left\{\frac{1}{p}\sum_{n=0}^{p-1} L_n\left[\frac{|z_1 - z_2|^2}{2l_0(B^*)^2}\right]\right\} \times \exp\left[-\frac{|z_1 - z_2|^2}{4l_0(B^*)^2}\right]\exp\left[\frac{z_1^* z_2 - z_1 z_2^*}{4l_0(B^*)^2}\right], \tag{13}$$

where $r_{12} = |z_1 - z_2|$ is the interparticle distance and the last term of equation (13) is merely a phase factor. Since the reduced magnetic field is given by $B^* = B/(2mp + 1)$ one notes that $l_p(r_{12})$ depends not only on p, but also on m. We can further simplify this expression by noting that $\sum_{n=0}^{p-1} L_n(x) = L_{p-1}^1(x)$, where $L_n^k(x) = (-1)^k\frac{\mathrm{d}^k}{\mathrm{d}x^k}(L_{n+k}(x))$ are the associated Laguerre polynomials of order $n = 0, 1, \ldots$ and degree $k = 0, 1, \ldots$

Since both electrons and CF-s have the same density, we can relate the CF magnetic length $l_0(B^*)$ to the true electronic magnetic length $l_0(B)$ by the simple relation, $l_0(B^*)^2 = l_0(B)^2(2mp+1)$, so we finally can write $l_p(r_{12})$ in terms of the natural dimensionless distance $r_{12}/l_0(B)$ as

$$l_p(r_{12}) = \left\{\frac{1}{p}L_{p-1}^1\left[\frac{1}{2(2mp+1)}\left(\frac{r_{12}}{l_0(B)}\right)^2\right]\right\} \times \exp\left[-\frac{1}{4(2mp+1)}\left(\frac{r_{12}}{l_0(B)}\right)^2\right] \times \exp\left[\frac{1}{4(2mp+1)}\frac{z_1^* z_2 - z_1 z_2^*}{l_0(B)^2}\right]. \tag{14}$$

For a non-zero value of m, both the exponential factors of equation (14) vanish in the $p \to \infty$ limit; using the formula $\lim_{p\to\infty} L_p^1(x/p)/p = J_1(2\sqrt{x})/\sqrt{x}$ we find that

$$l_{p\to\infty}(r_{12}) = 2\frac{J_1\left(\frac{1}{\sqrt{m}}\frac{r_{12}}{l_0(B)}\right)}{\left(\frac{1}{\sqrt{m}}\frac{r_{12}}{l_0(B)}\right)}, \tag{15}$$

where $J_1(x)$ is the Bessel function of the first order. We note that this expression corresponds to the statistical

exchange factor of a 2D system of fully-spin polarized ($g_s = 1$) free fermions whose Fermi radius is given by $k_{\rm F}(\nu = 1/(2m)) = 1/(\sqrt{m} l_0(B))$.

This proves that, for a system of fermions studied in the thermodynamic limit, the uncorrelated CF-s are exactly transformed into 2D spinless free fermions in the $p \to \infty$ limit, so that

$$\lim_{p\to\infty} \Psi^{\rm CF}_{\nu=p/(2mp+1)} = \Psi^{\rm Fermi}_{\nu=1/(2m)}, \qquad (16)$$

where the above Fermi wave function has the form

$$\Psi^{\rm Fermi}_{\nu=1/(2m)} = \hat{P}_{\rm LLL} \prod_{j<k}^{N} (z_j - z_k)^{2m} {\rm Det}\{\varphi_{\mathbf{k}}(\mathbf{r})\}, \qquad (17)$$

and $\varphi_{\mathbf{k}}(\mathbf{r}) - s$ are 2D normalized plane waves which fill a Fermi disk up to $|\mathbf{k}| \leq k_{\rm F}(\nu = 1/(2m))$. Based on these arguments we would expect the Fermi wave function to be a very good description for the even-denominator-filled state $\nu = 1/(2m)$. Indeed, the finite-size calculations of Rezayi and Read [11] have provided clear indications that this trial Fermi wave function is a very good ansatz for the ground state at $1/(2m)$.

The Rezayi-Read Fermi-sea-like wave function describes the even-denominator-filled state for both infinite (thermodynamic limit) and finite N. For finite number of particles, the CF wave function describes the filling state $\nu = p/(2mp + 1)$ with the largest finite p constrained by the size of system and finite number of particles. Depending on the finite values of N and Ω, the CF wave function can describe states very close to filling $\nu = 1/(2m)$, but not the exact filling $\nu = 1/(2m)$. In order to describe filling $\nu = 1/(2m)$ for a finite system of particles, one should use the original Rezayi-Read Fermi-sea-like wave function. Only in the thermodynamic limit and in the $p \to \infty$ limit, the CF wave function and the Rezayi-Read Fermi-sea-like wave function become identical and both describe the same even-denominator-filled state $\nu = 1/(2m)$. By describing the filling factor $\nu = 1/(2m)$ with the trial Fermi wave function $\Psi^{\rm Fermi}_{\nu=1/(2m)}$ we give the HLR approach in a more physical and microscopic version. The resulting theory is essentially equivalent [18] to the HLR theory, though in the HLR approach a transformation involving attaching delta-function fluxes to the electrons was employed.

3 Results and conclusions

In this paper we studied the CF wave function for a system of fermions in the thermodynamic limit and in particular its behaviour in a range of filling factors very close to the even-denominator-filled state $\nu = 1/(2m)$. We prove that in this limit the CF wave function $\Psi^{\rm CF}_{\nu=p/(2mp+1)}$ exactly transforms into the Rezayi-Read Fermi-sea-like wave function of equation (17) which is expected to be a very good description of the even-denominator-filled states. We applied the FHNC/0 theory to compute the radial distribution function and related quantities corresponding to the CF wave function as it approaches the even-denominator-filled state $\nu = 1/(2m)$. Although we do not include explicitly the projection operator and use an unprojected wave function, the intrinsic LL mixing of the CF wave function and the Jastrow factor already provide a good projection [19] onto the LLL, that should be particularly effective as far as ground-state properties are concerned. The interaction energy per particle was computed from the standard formula

$$u_\nu = \frac{1}{N} \frac{\langle \Psi^{\rm CF}_\nu | \hat{V} | \Psi^{\rm CF}_\nu \rangle}{\langle \Psi^{\rm CF}_\nu | \Psi^{\rm CF}_\nu \rangle} = \frac{\rho(\nu)}{2} \int {\rm d}^2 r_{12} [g_\nu(r_{12}) - 1] v(r_{12}), \qquad (18)$$

for ten filling factors $\nu = p/(2mp + 1)$ where $p = 1, 2, \ldots, 10$ and for $p \to \infty$. In particular we are interested in the series of filling factors with $m = 1$ which in the $p \to \infty$ limit ends with the half-filled state. As shown in Figure 1 the modulus squared statistical exchange factor $|l_p(r_{12})|^2$ corresponding to the uncorrelated state $\Phi_p(B^*)$ is not very different from the modulus squared exchange factor corresponding to the Slater determinant of plane waves at filling $\nu = 1/(2m)$ even for relatively small values of p.

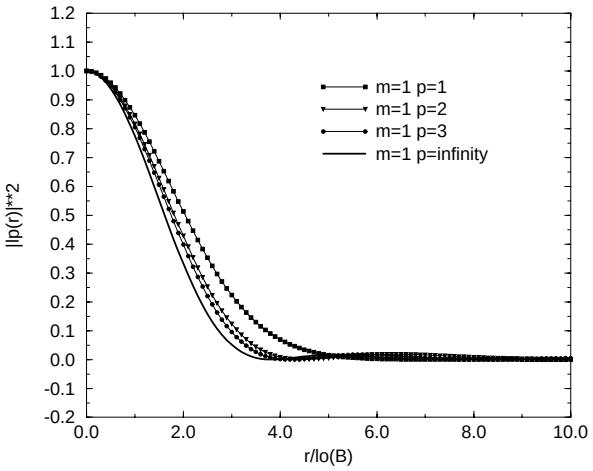

Fig. 1. We plot the modulus squared statistical exchange factor $|l_p(r_{12})|^2$ corresponding to the $\Phi_p(B^*)$ state for $p = 1, 2, 3$ and $m = 1$ and compare it with the analogous quantity corresponding to a 2D spinless system of free fermions at the corresponding density.

By increasing the value of p, as shown in Figure 2 the differences get extremely smaller and eventually vanish at filling factor $\nu = 1/(2m)$, where the state $\Phi_p(B^*)$ transforms into a Slater determinant of 2D spinless free fermions at the corresponding density. In Figure 3 we plot the radial distribution function corresponding to the first values of p for the series of filling factors $\nu = p/(2p + 1)$ and we compare them to the radial distribution function of the $\nu = 1/2$ state described by the Fermi wave function $\Psi^{\rm Fermi}_{\nu=1/(2m)}$ for $m = 1$. One clearly notes that the radial distribution function reveals more and more

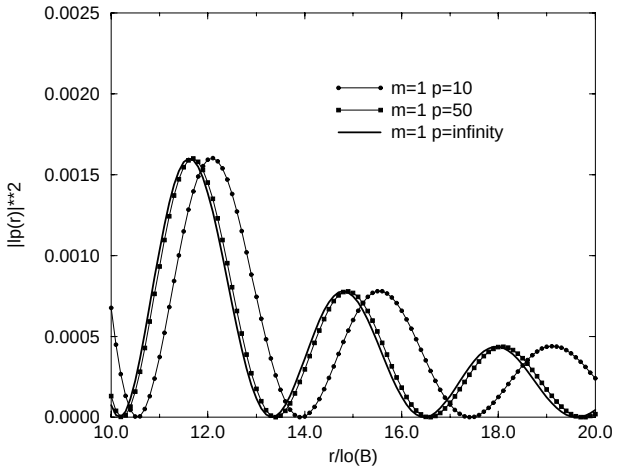

Fig. 2. For large values of p, the modulus squared statistical exchange factor $|l_p(r_{12})|^2$ corresponding to the $\Phi_p(B^*)$ state converges into the analogous quantity for a 2D spinless system of free fermions at a density corresponding to filling factor $\nu = 1/2$.

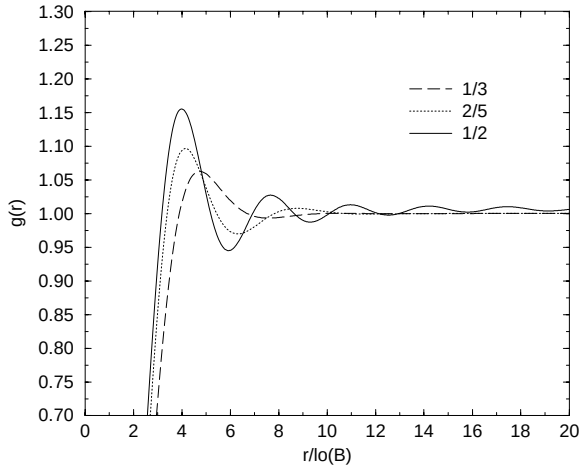

Fig. 3. The radial distribution function $g_\nu(r)$ for filling factors $\nu = p/(2p+1)$, $p = 1, 2, \ldots$ described by $\Psi^{\rm CF}_{\nu=p/(2p+1)}$ and for the half-filled state $\nu = 1/2$ state obtained as the $p \to \infty$ limit of such fillings and described by the $\Psi^{\rm Fermi}_{\nu=1/2}$ wave function. Calculations were performed with an unprojected wave function by using the FHNC/0 method.

pronounced peaks as the filling factor approaches the even-denominator value $1/(2m)$. This oscillatory behaviour has been seen [20] in Monte-Carlo calculations for filling factors in the vicinity of 1/2 (for instance filling 6/13) for both the projected and the unprojected CF wave function and our findings agree with that picture. Here we prove that the oscillations in the radial distribution function persist as $\nu \to 1/(2m)$ and get more robust at $\nu = 1/(2m)$ in agreement with our expectations. As seen from Figure 4, the initial oscillations for

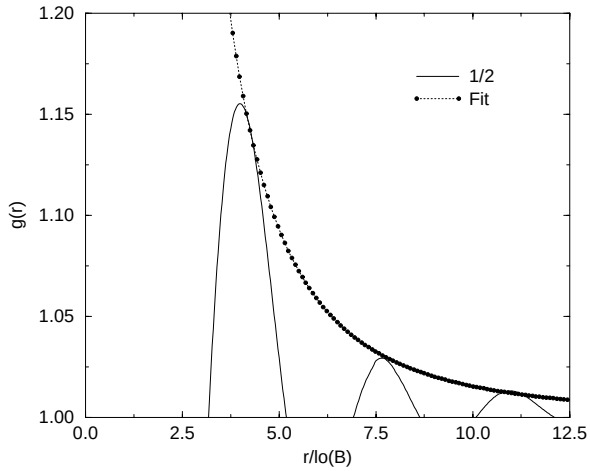

Fig. 4. The radial distribution function $g_{\nu=1/2}(r)$ for the half-filled state described by the unprojected Fermi-sea-like wave function $\Psi^{\rm Fermi}_{\nu=1/2}$ as a function of the dimensionless distance $r/lo(B)$. The oscillations are fitted reasonably well by the function $g_{\nu=1/2}(r) - 1 \propto \frac{\sin(2r/lo(B))}{(r/lo(B))^\alpha}$ with $\alpha \approx 2.6$. The envelope of $g_{\nu=1/2}(r) - 1$ has as a power-law decay.

$\nu = 1/2$ can be fitted reasonably well by the function $g_{\nu=1/2}(r) - 1 \propto \frac{\sin(2r/lo(B))}{(r/lo(B))^\alpha}$ with the parameter $\alpha \approx 2.6$. The presence of $2k_{\rm F}(\nu = 1/2)r$ oscillations and a power-law decay of the envelope provides clear evidence of Fermi-like behaviour. For small finite values of p there are only a few identifiable peaks of $g_\nu(r) - 1$, so an accurate analysis of the decay of the oscillations is very difficult to perform, however for larger values of p, for instance $p = 5$, 6, or 7, the same fitting function provides a rather good fit (with α being p-dependent) with the envelope decaying as a power-law also for such finite values of p.

Finite-size calculations for $N = 9$ spin-polarized (spinless) electrons confined to the LLL on a spherical surface [11] by Rezayi and Read confirm the existence of some sort of "Friedel-like" oscillations on the behaviour of $g(r)$ for filling $\nu = 1/(2m)$. The radial distribution function for the exact $N = 9$ ground state at half-filling has been found indistinguishable from that corresponding to the projected Fermi wave function suggesting that this wave function is a very good variational state. The FHNC/0 technique, which has the priority to treat in the thermodynamic limit the many-body wave function $\Psi^{\rm Fermi}_{\nu=1/(2m)}$, confirms and is in good qualitative agreement with this oscillatory behaviour. In Table 1 we display the values of the correlation energy per particle for ten states with filling factors $\nu = p/(2p+1)$ described by the unprojected CF wave function. From the results shown in Table 1 and from Figure 5 one notes that the correlation energy per particle $u_{\nu=p/(2mp+1)}$ for $m = 1$ smoothly converges to the $\nu = 1/2$ value (dotted line) in the $p \to \infty$ limit. If the results of Table 1 are quoted in units of $e^2/(4\pi\epsilon_0\epsilon R_0(\nu))$, where $R_0(\nu)$ is the ion disk radius given by $\pi R_0(\nu)^2 = 1/\rho(\nu)$, we find that $u_{\nu=p/(2p+1)} = Ce^2/(4\pi\epsilon_0\epsilon R_0(\nu))$, where C is

Table 1. The interaction energy per particle u_ν corresponding to the CF wave function $\Psi^{\rm CF}_{\nu=p/(2p+1)}$ for filling factors $\nu = p/(2p+1)$ where $p = 1, \ldots, 10$. In the $p \to \infty$ limit the above CF wave function exactly transforms into the Fermi wave function $\Psi^{\rm Fermi}_{\nu=1/2}$ which describes correlated 2D spinless fermions at half filling. The interaction energy is expressed in standard units of $e^2/(4\pi\epsilon_0\epsilon l_0(B))$ where $l_0(B)$ is the electronic magnetic length. The calculations were based on the FHNC/0 method using an unprojected wave function.

m	p	$\nu = p/(2mp+1)$	FHNC/0
1	1	1/3	−0.402(3)
1	2	2/5	−0.440(9)
1	3	3/7	−0.456(3)
1	4	4/9	−0.464(8)
1	5	5/11	−0.469(7)
1	6	6/13	−0.473(3)
1	7	7/15	−0.475(9)
1	8	8/17	−0.477(9)
1	9	9/19	−0.479(5)
1	10	10/21	−0.480(8)
1	∞	1/2	−0.492(1)

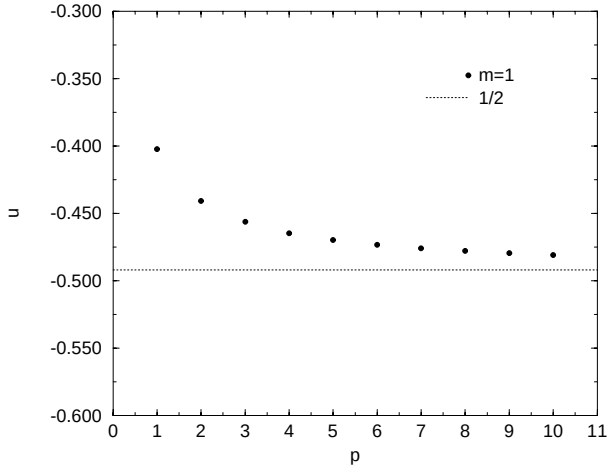

Fig. 5. The interaction energy per particle corresponding to the CF wave function $\Psi^{\rm CF}_{\nu=p/(2p+1)}$ (circle) for several values of p. In the $p \to \infty$ limit the wave function $\Psi^{\rm CF}_{\nu=p/(2p+1)}$ exactly transforms into the Fermi wave function $\Psi^{\rm Fermi}_{\nu=1/2}$ corresponding to the half-filled state (dotted line). The energies are expressed in standard units of $e^2/(4\pi\epsilon_0\epsilon l_0(B))$ and the FHNC/0 method was used to treat the unprojected wave functions.

roughly −1. Considering the more accurate Monte Carlo data of Kamilla *et al.* [21] obtained with the same unprojected wave function, one finds $C \approx -1.00(3)$. These results are highly suggestive to write

$$u_{\nu=p/(2p+1)} = C\frac{e^2}{4\pi\epsilon_0\epsilon R_0(\nu)} = C\sqrt{\frac{\nu}{2}}\frac{e^2}{4\pi\epsilon_0\epsilon l_0(B)}, \quad (19)$$

with

$$C = \sqrt{6}\frac{u_{\nu=1/3}}{\dfrac{e^2}{4\pi\epsilon_0\epsilon l_0(B)}}\cdot \quad (20)$$

If one considers $u_{\nu=1/3} = -0.410\frac{e^2}{4\pi\epsilon_0\epsilon l_0(B)}$ as the most accurate correlation energy value for filling 1/3 then the constant C should have the value $C = -1.0043$.

As a consequence of this, the radial distribution function corresponding to the unprojected CF wave function describing filling states $\nu = p/(2p+1)$ should satisfy the sum rule

$$\sqrt{\frac{\nu}{2}}\int \mathrm{d}\left(\frac{r}{l_o(B)}\right)\left[g_\nu\left(\frac{r}{l_o(B)}\right) - 1\right] = C \approx -1, \quad (21)$$

which we have not been able to prove analytically. If one uses fully projected CF wave functions [22] the quantity $u_{\nu=p/(2p+1)}/[e^2/(4\pi\epsilon_0\epsilon R_0(\nu))]$ is no more a constant, but is a monotonic function of p. These results can be intuitively understood by considering the difference in the short distance behaviours between various unprojected and projected functions. For all unprojected $\Psi^{\rm CF}_{\nu=2/5}$, $\Psi^{\rm CF}_{\nu=3/7}, \ldots$ the radial distribution function $g_\nu(r) \sim r^6$, as is also the case for $\Psi^{\rm CF}_{\nu=1/3}$. On the other hand, for trial wave functions fully projected within the LLL, it can be shown rigorously [23–25] that $g_\nu(r) \sim r^2$ for $\nu > 1/3$. With the plausible assumption that the interaction energy per particle is governed by the short distance behaviour of $g_\nu(r)$, it is clear that the fully LLL projected states at filling $2/5, 3/7, \ldots$ will have the correlation energy per particle in units of $e^2/(4\pi\epsilon_0\epsilon R_0(\nu))$ higher than the state at 1/3, but will have the same correlation energy as the state at 1/3 for the unprojected wave functions of equation (3).

In order to directly compare these results with the available finite-size calculations [11], it would be highly desirable to incorporate the LLL projection within the FHNC/0 scheme. The inclusion of the LLL projection operator immensely complicates the problem since in the fully LLL projected Hilbert space of Laughlin-like Jastrow wave functions, the plane wave $\varphi_{\mathbf{k}}(\mathbf{r}_j) = (1/\sqrt{\Omega})\exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r}_j)$ acts as an operator; namely z_j^* acts on a given function as $2\partial/\partial z_j$ and as a result the structure of a Slater determinant of single-particle orbitals is lost. One can adopt the projection technique by Bonesteel [26] used to calculate the excitation gaps of $\nu = 1/3, 1/5$ and $1/7$, which however is limited to Slater determinants spanning two Landau levels only. The extension of such a technique to more LL-s appears to be numerically unaccessible. A more general projection scheme, applied to few electron systems in a spherical geometry [27], seems to be more promising. Such a scheme brings in a many-body dependency on all single particle orbitals, which however can be handled by introducing state-dependent correlations in the wavefunction, in close analogy to "backflow" correlations [28] of liquid ^{3}He. By dropping the LLL constraint and considering an unprojected wave function we make accessible

the computation of several physical quantities without having the limitations of the exact calculations with few electrons where the extrapolation to the thermodynamic limit is not totally unambiguous.

To conclude, we prove that for systems of fermions studied in the thermodynamic limit the CF wave function exactly transforms into the Rezayi-Read Fermi-sea-like wave function (see Eq. (17)) in the $p \to \infty$ limit. We study the behaviour of the CF-s for a range of filling factors very close to the even-denominator-filled state and their evolution into a Fermi-like state at the even-denominator-filled state. These calculations give support to the picture of a robust Fermi-sea-like state of electrons at $\nu = 1/(2m)$.

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